NEW EVIDENCE ON THE IMPACT OF IMPLICIT TRADING COSTS ON ASSET PRICES IN THE RUSSIAN STOCK MARKET

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- The Russian stock market had already been known for its low liquidity before 2022, when it saw a significant outflow of foreign capital. Since its establishment in the early 1990s, the market has encountered several major liquidity crises, including those in the latter half of 1998 and in December 2014.
- The applicability of classical asset pricing models, such as the CAPM, may be limited in emerging capital markets due to several factors, including high costs associated with market impact and wide bid-ask spreads.
- The existing research on how market liquidity affects Russian stock prices is limited. However, illiquidity appears to be one of the most crucial factors contributing to the cheapness of Russian equities in terms of financial multiples.
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CONTRIBUTION

- Most previous research papers examining the liquidity premium have primarily focused on developed markets or emerging markets in Asia. The following characteristics of the Russian equity market provide an opportunity to approach this issue from a different perspective:
 - The Russian stock market is a centralized marketplace, unlike many developed and emerging markets. Since all trades are implemented in a consolidated limit-order book, it is possible to identify stock prices and trading volumes accurately.
 - The Moscow Exchange officials make frequent adjustments to the minimum lot sizes and minimum tick sizes. As a result, we can expect a limited and approximately equal Influence of such market frictions on illiquidity measures in the cross-section.
- We are the first to carry out the comparative analysis of two low-frequency volatility over volume liquidity proxies, the Amihud index (Amihud, 2002) and the liquidity ratio developed by Kyle and Obizhaeva (2016), and check the conjectures related to differences between them following modern market microstructure theories.

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PRIOR LITERATURE - I

- I. Market liquidity and asset prices
 - Developed markets: Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Chordia and Swaminathan (2000), Amihud (2002) found significant cross-sectional and/or time-series relationships between market liquidity and US stock returns (illiquidity premia).
 Halka (2001), Hasbrouck and Seppi (2001), Acharya and Pedersen (2005) demonstrate different sources of illiquidity risk.
 Ben-Rephael (2015): the liquidity premium in the U.S. stock market has declined over the past few years; liquidity is now priced only for the smallest stocks.
 - Emerging and frontier markets: Bekaert, Harvey, and Lundblad (2007): intrinsic illiquidity remains a significant factor for expected stock returns despite globalization.
 - Cakici and Zaremba (2021): illiquidity premium exists only among microcap stocks.
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 - Russian market: Teplova and Mikova (2014a) and Teplova and Mikova (2014b): momentum strategies in the Russian stock market can be enhanced by implementing better liquidity control. 10 / 26

PRIOR LITERATURE - II

II. Stock liquidity measures

- <u>Multifaceted liquidity</u>: Kyle (1985) shows that liquidity has several
 different dimensions (depth, tightness, and resiliency). Cochrane (2004):
 it is difficult to give a correct definition of market liquidity and find the
 direct impact of implicit trading costs on asset prices.
- Low-frequency proxies for market liquidity:
 - Amihud illiquidity measure, which is the daily ratio of absolute stock return to its dollar volume, is one of the most popular proxies for market impact costs. Goyenko, Holden, and Trzcinka (2009), Hasbrouck (2009): the Amihud measure is one of the best low-frequency proxies for Kyle's lambda, the standard measure of market depth.
 - volume illiquidity proxy, the invariance-implied illiquidity ratio.

 Kyle and Obizhaeva (2018): this measure is theoretically related to market impact costs, relative bid-ask spread, market resiliency, pricing accuracy, and funding illiquidity.

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KYLE AND OBIZHAEVA RATIO VS AMIHUD INDEX

Both measures are volatility over volume asset-specific illiquidity proxies: The Kyle and Obizhaeva (invariance-implied) ratio is $1/L_{KO} = (\frac{\sigma_{it}^2}{V_{it}})^{1/3}$; The Amihud ratio is $1/L_{Amihud} = \frac{|\sigma_{it}|}{V_{it}}$, where σ_{it} is the return volatility and V_{it} is monetary (e.g., dollar or ruble) trading volume for stock i measured over time period t.

- Unlike 1/L_{Amihud}, the invariance-implied ratio 1/L_{KO} satisfies two fundamental principles of finance: 1) the time-clock irrelevance principle (the trading game is independent of the time clock) and 2) the Modigliani-Miller irrelevance principle (the trading game involving a financial security issued by a firm is independent of its capital).
- The Amihud ratio 1/L_{Amihud} is derived from the assumption that the standard deviation of order imbalances is proportional to volume, which seems to be unrealistic.
- The Amihud ratio is related to another unreasonable conjecture: all stocks have the same number of meta-orders - bets executed over long intervals to minimize trading costs per unit of time. The Kyle and Obizhaeva ratio 1/L_{KO} is based on the more realistic assumption: the expected arrival of meta-orders varies across stocks and depends on trading activity.

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ILLUSTRATION

Theoretical reasoning lead to the following implications.

	$1/L_{Amihud}$	$1/L_{KO}$
Panel A. Liquidity		
Actively traded stocks	Overestimated	Correctly estimated
Inactively traded stocks	Underestimated	Correctly estimated
Panel B. Illiquidity premium		
Actively traded stocks	Underestimated	Correctly estimated
Inactively traded stocks	Overestimated	Correctly estimated

HYPOTHESES

We propose and test the following hypotheses implying certain predictions about the relationship between illiquidity and asset prices in the cross-section and in the time-series.

- **Hypothesis 1**: The slope coefficient on $1/L_{Amihud}$ and $1/L_{KO}$ is positive and significant in the cross-sectional regressions with additional regressors that underlie **Fama and French (1993)** factor portfolios (beta, size, and book-to-market).
- Hypothesis 2: Within the cross-section framework, the statistical significance of the slope coefficient on the Amihud index for the subsamples of actively traded large-cap (non-actively traded small-cap) stocks is lower (higher) than the statistical significance of the corresponding slope coefficient on the invariance-implied ratio.
- Hypothesis 3: Zero-investment portfolios that are long in illiquid stocks and short in liquid stocks earn significant abnormal returns that are not explained by Carhart (1997) factors (beta, size, book-to-market, and momentum). The effect is stronger for small-capitalization stocks, and returns of small-stock (large-stock) portfolios are higher (lower) when the Amihud index is used as a proxy for illiquidity.

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DATA

- Sample: 241 ordinary and preferred Russian stocks traded on the Moscow Exchange
 Time frame: January 2010 - December 2020 (21,023 stock-month observations after applying all data cleaning procedures)
- Databases: Thomson Reuters Eikon, Laboratory for Analysis of Institutions and Financial Markets (RANEPA)
 - Trade-level daily data (trading volume in the number of stocks, open and close prices) to estimate illiquidity ratios and beta coefficients, the logarithm of market capitalization;
 - Monthly book-to-market ratios;
 - Monthly time series on Carhart (1997) factors.

EMPIRICAL METHODOLOGY

• To test **Hypotheses 1** and **2**, we use the Fama and MacBeth (1973) framework:

$$R_{i,m} = \alpha_0 + \alpha_1 \Lambda_{i,m-1} + (\alpha_2 BETA_{i,m-1} + \alpha_3 BM_{i,m-1} + \alpha_4 SIZE_{i,m-1}) + \epsilon_{i,m},$$
(1)

where $R_{i,m}$ is the monthly excess stock return in month m; $\Lambda_{i,m-1}$ is either $1/L_{Amihud}$ or $1/L_{KO}$ in month m-1; $BETA_{i,m-1}$ is the stock market beta in month m-1; $BM_{i,m-1}$ is the book-to-market ratio in the end of month m-1; $SIZE_{i,m-1}$ is the logarithm of market capitalization at the end of month m-1.

The Newey and West (1987) procedure with three lags is used to take into account possible autocorrelation and heteroscedasticity.

• To test **Hypothesis 3**, we calculate the time-series averages of monthly equal-weighted returns of 10 zero-investment portfolios, which are long in illiquid stocks and short in liquid stocks but size-neutral. (The Amihud index and the Kyle and Obizhaeva ratio are used separately as illiquidity measures.) Finally, we regress monthly returns of zero-investment portfolios on Carhart (1997) factors and compare alpha coefficients across portfolios.

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where $R_{i,m}$ is the monthly excess stock return in month m; $\Lambda_{i,m-1}$ is either $1/L_{Amihud}$ or $1/L_{KO}$ in month m-1; $BETA_{i,m-1}$ is the stock market beta in month m-1; $BM_{i,m-1}$ is the book-to-market ratio in the end of month m-1; $SIZE_{i,m-1}$ is the logarithm of market capitalization at the end of month m-1.

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MAIN RESULTS

- We cannot completely reject Hypothesis 1 since we have found statistical significance for small-cap stocks and the 2014–2020 subsample in univariate regressions. At the same time, we reject Hypothesis 2 since we have not fixed a cross-sectional pattern of overperformance of illiquid stocks.
- Carhart (1997) factors do not completely explain portfolio returns of small-stock portfolios. The positive returns of composite zero-investment portfolios are also unexplained by these factors. In addition, the Amihud measure does overestimate the illiquidity premium for inactively traded small-capitalization stocks, as predicted by the invariance hypothesis.

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APPENDIX I. TESTS ON ILLIQUIDITY PORTFOLIOS

our-factor regressions. small caps (non-actively traded st				d stocks)		large caps (actively traded stocks)					
Panel A. (a proxy: 1/L ^{KO})		_							_		
Coeff.\ Decile	Low	2	3	4	5	6	7	8	9	High	All
а	2.69**	2.88***	0.57	1.26*	0.69	-0.08	- 0.53	0.28	- 0.66°	-0.01	0.71**
	(2.26)	(2.97)	(0.70)	(1.63)	(1.09)	(-0.13)	(-0.87)	(0.38)	(-1.55)	(-0.02)	(2.30)
β_l	0.44**	0.15	0.07	-0.11	0.31**	0.28*	0.27**	-0.20	0.15**	-0.08	0.13**
	(1.84)	(0.62)	(0.36)	(-0.85)	(2.03)	(1.41)	(1.47)	(-1.12)	(1.80)	(-0.93)	(2.15)
Sį	0.41**	0.01	0.41**	0.10	0.20	0.24	0.22**	-0.05	0.30***	-0.03	0.18**
	(1.67)	(0.03)	(1.98)	(0.45)	(0.85)	(1.05)	(1.34)	(-0.28)	(3.34)	(-0.30)	(2.28)
	-0.16	-0.37*	0.31*	-0.05	- 0.05	-0.04	0.05	0.02	0.10	-0.09	-0.03
	(-0.64)	(-1.41)	(1.59)	(-0.36)	(-0.29)	(-0.25)	(0.44)	(0.12)	(0.87)	(-0.86)	(-0.40)
m _I 0.23 (1.14)	0.23	- 0.13	0.01	- 0.31	0.00	0.18*	-0.04	0.05	0.09	0.16***	0.02
	(1.14)	(-0.63)	(0.04)	(-2.20)	(0.01)	(1.39)	(-0.22)	(0.49)	(1.18)	(2.72)	(0.44)
Adj. R-squared	0.002	- 0.009	0.026	0.013	-0.004	0.005	-0.004	- 0.017	0.027	0.028	0.024
GRS-test: p-value=0.0041											
Panel B. (a proxy: 1/L ^{Amthud})											
Coeff.\ Decile	Low	2	3	4	5	6	7	8	9	High	All
a	3.60***	3.27***	1.32*	1.82***	0.69	-0.37	-0.78*	0.02	-0.47	-0.04	0.91***
	(3.20)	(3.37)	(1.37)	(2.53)	(1.06)	(-0.55)	(-1.31)	(0.03)	(-1.01)	(-0.09)	(2.67)
β_l	0.17	0.22	0.08	-0.11	0.34**	0.354**	0.26*	-0.18	0.06	-0.10	0.11**
(0	(0.79)	(0.95)	(0.46)	(-0.88)	(2.07)	(1.97)	(1.59)	(-1.05)	(0.66)	(-1.16)	(1.76)
s ₁ 0.25 0.12 (1.04) (0.56)	0.12	0.40**	0.08	0.25	0.36**	0.19	-0.03	0.26***	-0.03	0.19**	
	(0.56)	(2.01)	(0.39)	(1.02)	(1.79)	(1.24)	(-0.16)	(2.56)	(-0.35)	(2.04)	
h_l $-0.08 -0.30 0.34^*$ $(-0.27) (-1.27) (1.61)$	0.34*	- 0.08	- 0.03	- 0.02	0.10	0.02	0.10	- 0.04	0.00		
	(1.61)	(-0.63)	(-0.16)	(-0.14)	(0.75)	(0.10)	(0.71)	(-0.52)	(0.01)		
m _I 0.15 - 0.13 (0.68) (-0.64)	- 0.06	- 0.25	0.06	0.20**	- 0.08	0.04	0.03	0.20***	0.02		
	(-0.64)	(-0.41)	(-1.91)	(0.54)	(1.70)	(-0.50)	(0.42)	(0.32)	(3.30)	(0.25)	
Adi, R-squared	- 0.022	- 0.023	0.014	0.004	0.002	0.030	0.004	- 0.018	0.006	0.050	0.007
GRS-test: p-value=0.0031											

For each portfolio i, we perform the following regression.

(6)(6)

 $R_i = a_i + \beta_i RMRF + s_i SBM + h_i HML + m_i MOM + \epsilon_i$

Where A is the Amihud ratio or the invariance-implied ratio, BEA is the stock market beta; BM is the book-to-market ratio, SIZE is the logarithm of market capitalization. R^2 overall is $1 - (1 - corr^2[\hat{R}_R, R_R]) - \frac{n-1}{R_R-1}$, where n is a number of observations, p is a number of regressors. The columns 2–5 report the results for five portfolios that are monthly formed based on trading volume in a prior month. The last two columns show the results of the cross-sectional regressions for two subperiods: January 2010 - February 2014 and March 2014 - December 2020. The numbers in brackets are (Newey and West 1987) adjusted - statistics with 3 lass. The asterisks **, **, and **, ** denote statistical significance at the 10% and 1% levels. respectively.

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APPENDIX II. FURTHER RESEARCH

- Gurov (2023): unexpected illiquidity shocks have significant negative impact on contemporaneous returns of Russian stocks over 2010-2020; premium for expected illiquidity is negligible.
- Working paper: the invariance-implied transaction cost model to assess the implicit costs of factor strategies that require frequent rebalancing (momentum or reversal). The main research question: what is the optimal frequency of rebalancing?

Market microstructure invariance theory (**Kyle and Obizhaeva**, **2016**): For orders of a given percentage of average daily volume, bid-ask spread is a relatively larger component of transactions costs for less active stocks, and market impact is a relatively larger component of costs for more active stocks.