Principal components of stock market dynamics

Methodology and applications in brief
(to be updated…)

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# Why principal components are needed

Objectives

- understand the evidence of more than one factor driving the stock market, market factor should be supplemented with others, possibly less significant

- identify set of factor components driving stock market and its basic segments

- set up multi-factor model that is better than one-factor for monitoring of market dynamics, hedge, pricing, risk-valuation

Similar to modeling the shape of the yield curve

Modelling the shape of the yield curve – very similar process

• evidence of more than one factor driving the term structure

• principal components of dynamics are needed to hedge the shape of the yield curve in practice

Yield curve as at 9th February 2005 for USD
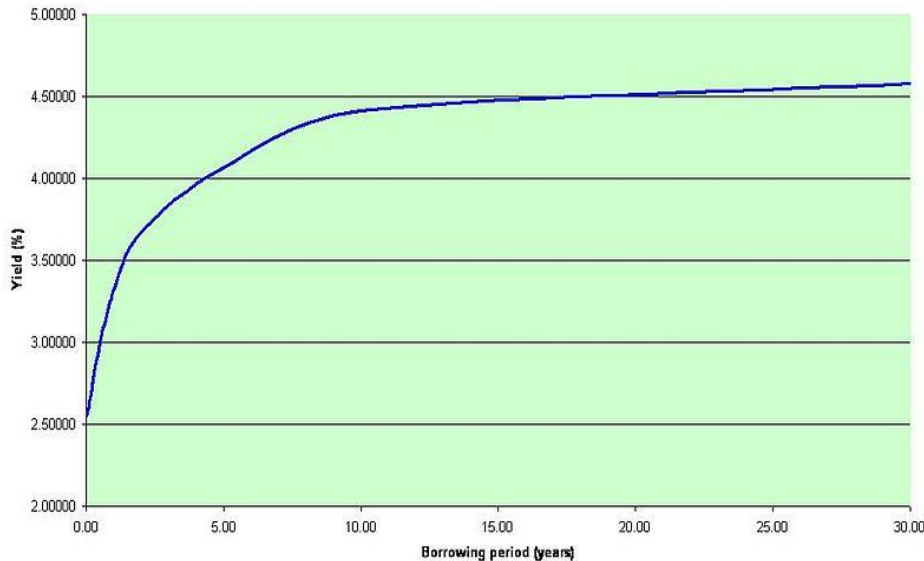
The folowing three factors are used: level, slope, curvature
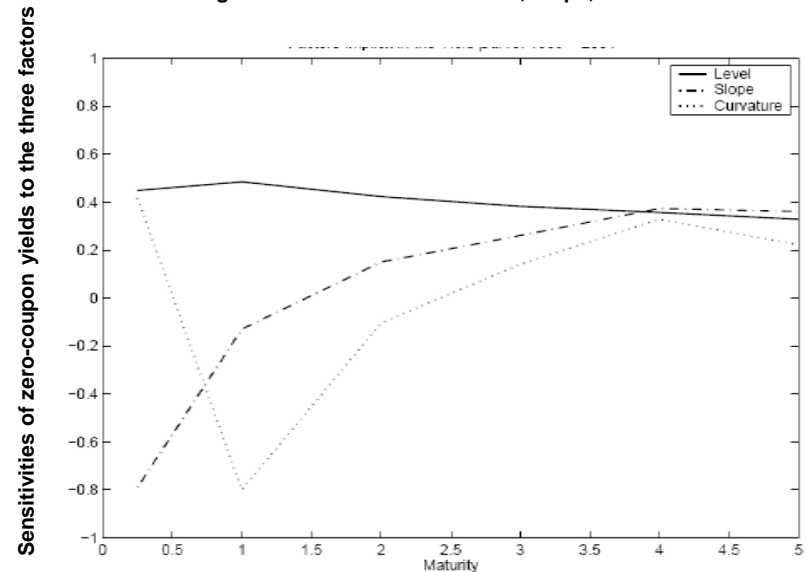
In reality, do we have only one factor?

Correlation between USA stock market portfolios formed on size. Data: Fama-French decile portfolios' returns formed on market equity, <u>based on monthly returns</u>, 1926-2010

	LO10	DEC2	DEC3	DEC4	DEC5	DEC6	DEC7	DEC8	DEC9	HI10
LO10	1,00	0,96	0,93	0,91	0,88	0,86	0,85	0,81	0,79	0,70
DEC2		1,00	0,97	0,97	0,95	0,92	0,91	0,88	0,86	0,78
DEC3			1,00	0,98	0,97	0,96	0,94	0,92	0,90	0,82
DEC4				1,00	0,98	0,97	0,96	0,94	0,92	0,84
DEC5					1,00	0,98	0,97	0,96	0,94	0,87
DEC6						1,00	0,98	0,97	0,96	0,90
DEC7							1,00	0,98	0,97	0,91
DEC8								1,00	0,98	0,93
DEC9									1,00	0,95
HI10										1,00

• Correlation matrix indicates that difference in return dynamics is increasing by the difference in market equity.

• Are there underlying factors that influence different portfolios in different manner?

In reality, do we have only one factor?

Correlation between Russia stock market portfolios formed on size. Data: MICEX capitalisation indices, based on <u>daily price levels (left) and daily returns (right)</u>, 2005-2011

	BASE	STANDARD	HIGH
BASE	1	0,97	0,92
STANDARD		1,00	0,96
HIGH			1,00

	BASE	STANDARD	HIGH
BASE	1,00	0,75	0,66
STANDARD		1,00	0,82
HIGH			1,00

- Correlation matrix indicates that difference in price dynamics is increasing by the difference in market equity.

- Are there underlying factors that influence different portfolios in different manner?

Identifying principal components*

- Let us find 3 factors that explain the dynamics of the ME portfolios

  - The analysis for more than 3 factors is a straightforward extension

- Assume the following dynamics of portfolios returns

$$r_1(t) = \alpha_1 + \beta_{11}\Delta\phi_1(t) + \beta_{12}\Delta\phi_2(t) + \beta_{13}\Delta\phi_3(t) + \varepsilon_1(t)$$

$$r_2(t) = \alpha_2 + \beta_{21}\Delta\phi_1(t) + \beta_{22}\Delta\phi_2(t) + \beta_{23}\Delta\phi_3(t) + \varepsilon_2(t)$$

...

$$r_n(t) = \alpha_n + \beta_{n1}\Delta\phi_1(t) + \beta_{n2}\Delta\phi_2(t) + \beta_{n3}\Delta\phi_3(t) + \varepsilon_n(t)$$

meaning that each factor impacts all of the returns with sensitivities β (alternatively the same regression for price levels can be used).
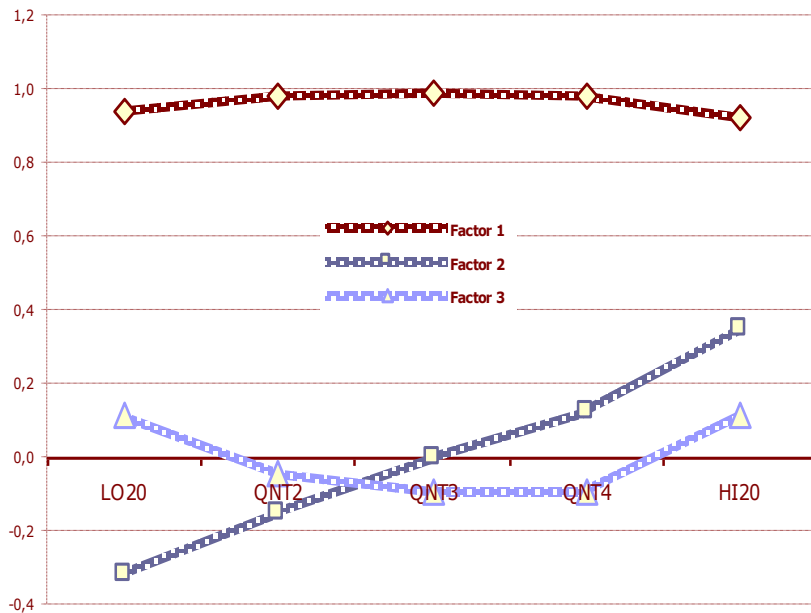
Principal component analysis

- We look for factors that are implicit in the movements over time of the various returns

- To determine them we are going apply *Principal Component Analysis*

  - Finding first principal component means that we are maximizing a weighted average of the R-squares of the above regressions

  - After computing the first component, we can find the residuals, applying them instead of returns, and find the second factor, and so on

Principal components in practice (USA stock market)

- Using monthly returns for the US stock market between 1926 and 2000, we have the following sensitivities of size-based portfolios returns to the three factors
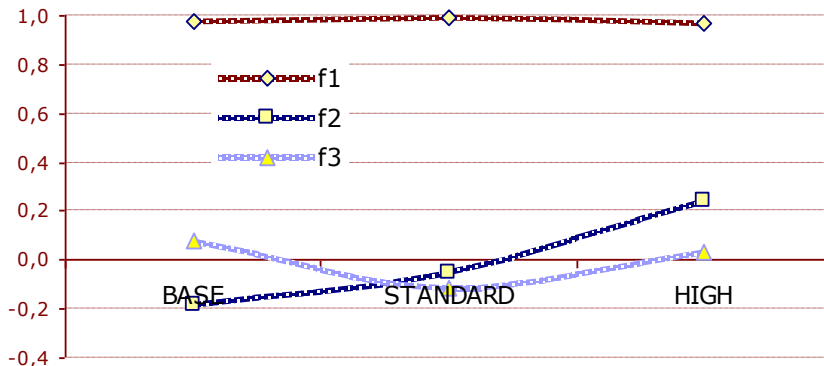

Correlation matrix

	f1	f2	f3
LO20	0,94	–0,314	0,12
QNT2	0,98	–0,151	–0,04
QNT3	0,99	–0,003	–0,09
QNT4	0,98	0,124	–0,09
HI20	0,93	0,350	0,12

Extraction Method: Principal Component Analysis.

Principal components in practice (Russia stock market)

Using daily price data for the Russia stock market between 2005 and 2011, we have the following sensitivities of size-based portfolios' price level (1) and returns (2) to the three factors

Correlation matrix, price levels

	f1	f2	f3
BASE	0,98	-0,19	0,08
STANDARD	0,99	-0,05	-0,11
HIGH	0,97	0,24	0,04
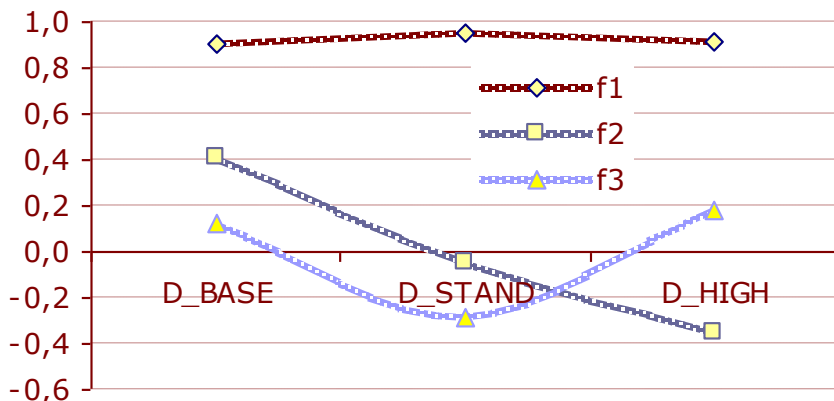
Correlation matrix, returns

|	f1	f2	f3
D_BASE	0,90	0,41	0,12
D_STAND	0,96	-0,05	-0,29
D_HIGH	0,92	-0,35	0,18

Empirical components' description

- First factor has roughly the same effect on all portfolio yields/prices. For this reason, it can be referred to as **parallel shift/level.** It explains most of the variation

- Second factor has negative impact on small stocks and positive impact on big stocks. This is a **slope** factor

- The third factor is positive at the small and big stocks, and negative at the medium stocks. It can be referred to as **curvature**

Returns or price levels?

- Applying factor analysis to returns or price levels should lead to the same results

  - Meaning "return factors" can be transformed mathematically into "level factors" correspondingly and vice versa

- Empirically there are some difficulties for return data

  - Positive correlation in levels with slightly lagged data can lead to negative correlation in returns

  - Daily returns are very "noisy", for weekly or monthly returns a long period of data is needed

- For the above reasons *price level* components have been extracted for Russia stock market
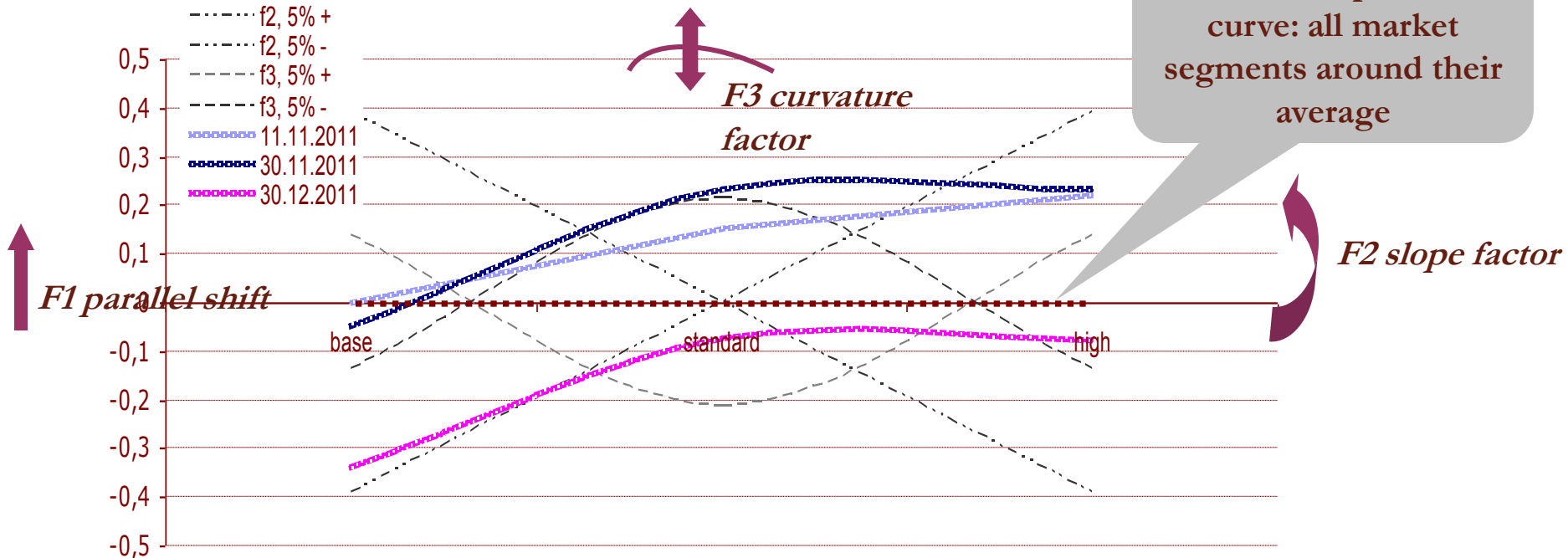
Principal components in dynamics

- Three factor component in dynamic, Russia stock market (based on daily price levels)

Factor dynamics okt.2011-feb.2012гг.
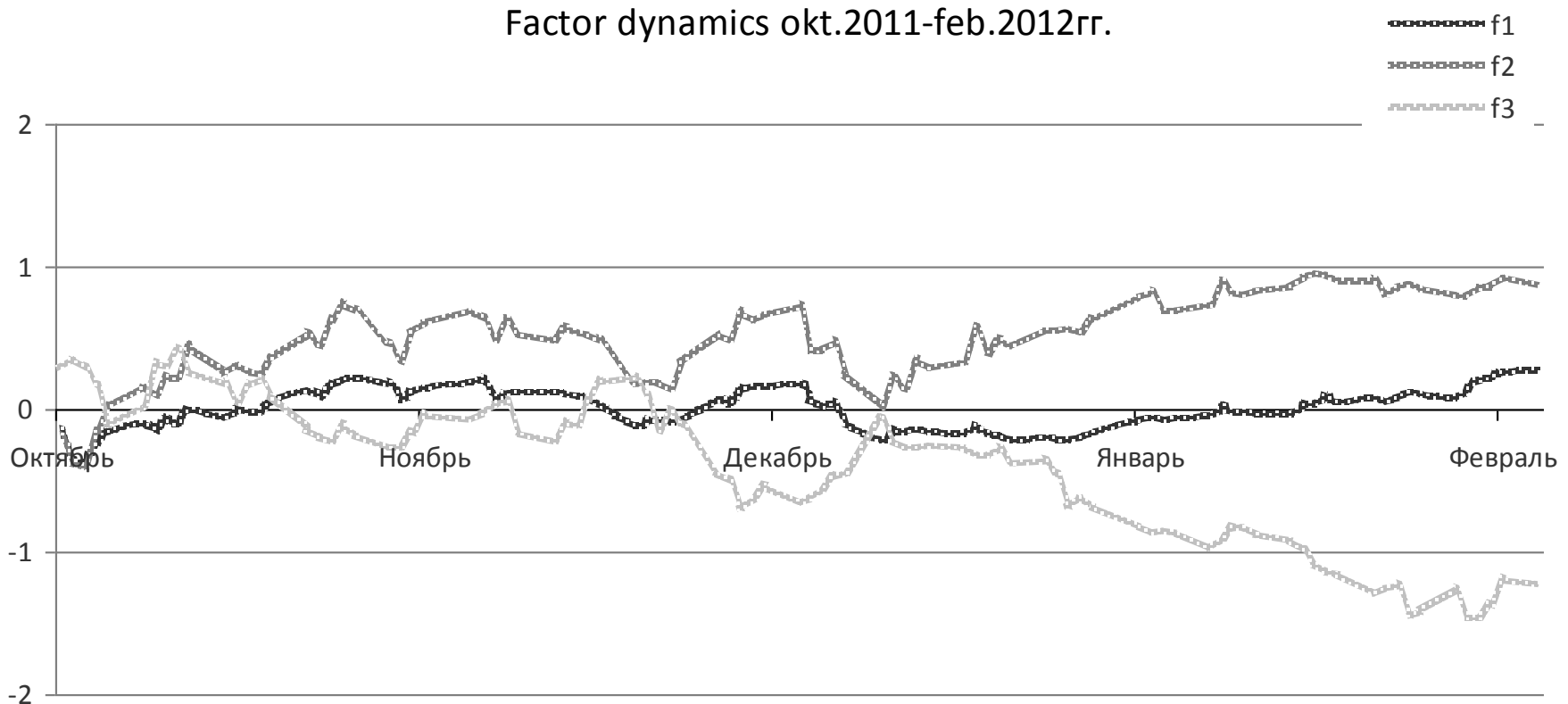
Constructing "market curve"

- Based on three factor components we can construct "stock market curve" which indicates underpriced/overpriced segments.

Market curve, russian stock market - micex

Legend:
- f2, 5% +
- f2, 5% -
- f3, 5% +
- f3, 5% -
- 11.11.2011
- 30.11.2011
- 30.12.2011

F3 curvature factor

F1 parallel shift

F2 slope factor

Normal shape of the curve: all market segments around their average

base   standard   high

Based on Russia stock market data (daily price levels), 2005-2011.

Application – multifactor asset pricing model (1)

- Expected return of any marketable security can be written as a function of the expected return of the efficient portfolio

$$E[r_i] = r_f + \beta_i^{eff} * (E[r_{eff}] - r_f)$$

- It is extremely difficult to identify portfolios that are efficient but we can use collection of portfolios from which the efficient portfolio can be constructed to measure risk

- Each factor (portfolio) captures different components of the systematic risk

- Multifactor models is significant improvement over the CAPM, it is widely used in academic literature and in practice to measure risk and to calculate cost of capital

Application – multifactor asset pricing model (2)

- The above described factors represent the following portfolios

  - The **level factor** is a market portfolio (to the extent it can be derived from stock market data)

  - The **slope factor** is small-minus-big portfolio. It is similar (but not identical) to SMB factor from Fama-French model).

  - The **curvature factor** is "small and big minus medium" portfolio.

- Taking into account the set of 3 factors the multifactor models becomes

$$E[r_i] = r_f + \beta_i^{f1} * (E[r_{f1}] - r_f) + \beta_i^{f2} * (E[r_{f2}] - r_f) + \beta_i^{f3} * (E[r_{f3}] - r_f)$$

Application – multifactor asset pricing model (3)

- Most popular multifactor model specification by now is based on market, small-minus-big and high-minus-low factors (see http://en.wikipedia.org/wiki/Fama%E2%80%93French_three-factor_model):

$$E[r_i] = r_f + \beta_i^{Mkt} * (E[r_{Mkt}] - r_f) + \beta_i^{SMB} * (E[r_{SMB}] - r_f) + \beta_i^{HML} * (E[r_{HML}] - r_f)$$

- **But these factors were selected empirically whereas level, slope and curvature factors <u>are derived mathematically</u> (using principal components analysis)**

Further development

- Factor components methodology and components' values are published on http://fmlab.hse.ru/

- Ready for any discussion and cooperation with individuals/academics/financial instituties of further development and calculation of the presented factor components in form of stock indices or returns.

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